Appendix G

Derivation of the Scattered Fields

in the Medium 1 at Air and Burnt Coal Seam Interface

From (5.27), the vector nature of $\boldsymbol{E}^s$ is characterized by

$$\begin{aligned}\boldsymbol{E}_n^s &\equiv \boldsymbol{E}^s / KI_1 \\ &= \hat{\boldsymbol{n}}_{so} \times \left(\hat{\boldsymbol{n}}_1 \times \boldsymbol{E} - h_1 \hat{\boldsymbol{n}}_{so} \times \left(\hat{\boldsymbol{n}}_1 \times \boldsymbol{H}\right)\right) \\ &= \hat{\boldsymbol{n}}_{so} \times \left(\hat{\boldsymbol{n}}_1 \times \boldsymbol{E}\right) + \left(h_1 \hat{\boldsymbol{n}}_1 \times \boldsymbol{H}\right) - \left(\hat{\boldsymbol{n}}_{so} \cdot \left(h_1 \hat{\boldsymbol{n}}_1 \times \boldsymbol{H}\right)\right)\hat{\boldsymbol{n}}_{so}\end{aligned} \tag{G.1}$$

The polarization factors which characterize different polarization states are

$$\hat{\boldsymbol{h}}_{so} \cdot \boldsymbol{E}_n^s = \hat{\boldsymbol{v}}_{so} \cdot \left(\hat{\boldsymbol{n}}_1 \times \boldsymbol{E}\right) + \hat{\boldsymbol{h}}_{so} \cdot \left(h_1 \hat{\boldsymbol{n}}_1 \times \boldsymbol{H}\right) \tag{G.2}$$

$$\hat{\boldsymbol{v}}_{so} \cdot \boldsymbol{E}_n^s = \hat{\boldsymbol{v}}_{so} \cdot \left(h_1 \hat{\boldsymbol{n}}_1 \times \boldsymbol{H}\right) - \hat{\boldsymbol{h}}_{so} \cdot \left(\hat{\boldsymbol{n}}_1 \times \boldsymbol{E}\right) \tag{G.3}$$

Under the stationary-phase approximation, $\hat{\boldsymbol{n}}_1$, $\hat{\boldsymbol{t}}$, $\hat{\boldsymbol{d}}$ can be expressed in terms of propagation vectors, $\hat{\boldsymbol{n}}_{so}$ and $\hat{\boldsymbol{n}}_i$, as

$$\begin{aligned}\hat{\boldsymbol{n}}_1 &= \frac{-\hat{\boldsymbol{x}}Z_x - \hat{\boldsymbol{y}}Z_y + \hat{\boldsymbol{z}}}{\left(1 + Z_x^2 + Z_y^2\right)^{\frac{1}{2}}} = \frac{\hat{\boldsymbol{x}}q_x + \hat{\boldsymbol{y}}q_y + \hat{\boldsymbol{z}}q_z \left|q_z\right|}{q_z q} \\ &= \frac{k_1\left(\hat{\boldsymbol{n}}_{so} - \hat{\boldsymbol{n}}_i\right)\left|q_z\right|}{q_z q}\end{aligned} \tag{G.4}$$

where $q^2 = q_x^2 + q_y^2 + q_z^2 = 2k_1^2\left(1 - \left(\hat{\boldsymbol{n}}_{so} \cdot \hat{\boldsymbol{n}}_i\right)\right)$

$$\begin{aligned}\hat{\boldsymbol{t}} &= \frac{\hat{\boldsymbol{n}}_i \times \hat{\boldsymbol{n}}_1}{\left|\hat{\boldsymbol{n}}_i \times \hat{\boldsymbol{n}}_1\right|} = \frac{\left(\hat{\boldsymbol{n}}_1 \times \hat{\boldsymbol{n}}_{so}\right)\left|q_z\right|}{q_z D_1} \\ &= \frac{\left(\hat{\boldsymbol{x}}q_y \cos q_o - \hat{\boldsymbol{y}}\left(q_x \cos q_o + q_z \sin q_o\right) + \hat{\boldsymbol{z}}q_y \sin q_o\right)}{q_z k_1^2 D_1}\end{aligned} \tag{G.5}$$

where

$$D_1 = |\hat{\boldsymbol{n}}_i \times \hat{\boldsymbol{n}}_s| = \left| (\hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{v}}_{so})^2 + (\hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{h}}_{so})^2 \right|^{\frac{1}{2}} \tag{G.6}$$

and

$$\begin{aligned} \hat{\boldsymbol{d}} &= \hat{\boldsymbol{n}} \times \hat{\boldsymbol{t}} = ((\hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{n}}_{so})\hat{\boldsymbol{n}}_i - \hat{\boldsymbol{n}}_{so})|q_z|/(q_z D_1) \\ &= \frac{-(\hat{\boldsymbol{x}} \cos q_o (q_x \cos q_o + q_z \sin q_o) + \hat{\boldsymbol{y}} q_y + \hat{\boldsymbol{z}} \sin q_o (q_x \cos q_o + q_z \sin q_o))}{q_z k_1^2 D_1} \end{aligned} \tag{G.7}$$

with these local coordinate vectors expressed in $\hat{\boldsymbol{n}}_i$ and $\hat{\boldsymbol{n}}_{so}$, all other vector products can be expressed in $\hat{\boldsymbol{n}}_i$ and $\hat{\boldsymbol{n}}_{so}$

$$\hat{\boldsymbol{n}}_1 \times \hat{\boldsymbol{t}} = (\hat{\boldsymbol{n}}_i + \hat{\boldsymbol{n}}_{so})(1 - (\hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{n}}_s))k_1/(qD_1) \tag{G.8}$$

$$\hat{\boldsymbol{n}}_1 \cdot \hat{\boldsymbol{n}}_i = -k_1\{1 - (\hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{n}}_s)\}|q_z|/(q_z q) = -q|q_z|/(2k_1 q_z) = -\cos q_o \tag{G.9}$$

$$\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{t}} = (\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}_{so})/\overline{D}_1 \tag{G.10}$$

$$\hat{\boldsymbol{h}}_{\boldsymbol{so}} \cdot \hat{\boldsymbol{t}} = -(\hat{\boldsymbol{v}}_{\boldsymbol{so}} \cdot \hat{\boldsymbol{n}}_i)/\overline{D}_1 \tag{G.11}$$

$$\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{t}} = -(\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{n}}_{so})/\overline{D}_1 \tag{G.12}$$

$$\hat{\boldsymbol{v}}_{\boldsymbol{so}} \cdot \hat{\boldsymbol{t}} = -(\hat{\boldsymbol{h}}_{\boldsymbol{so}} \cdot \hat{\boldsymbol{n}}_i)/\overline{D}_1 \tag{G.13}$$

$$\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{d}} = -(\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{n}}_{so})/\overline{D}_1 \tag{G.14}$$

$$\hat{\boldsymbol{h}}_{\boldsymbol{so}} \cdot \hat{\boldsymbol{d}} = (\hat{\boldsymbol{h}}_{\boldsymbol{so}} \cdot \hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{n}}_{so} \cdot \hat{\boldsymbol{n}}_i)/\overline{D}_1 \tag{G.15}$$

$$\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{d}} = -(\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}_{so})/\overline{D}_1 \tag{G.16}$$

$$\hat{\boldsymbol{v}}_{\boldsymbol{so}} \cdot \hat{\boldsymbol{d}} = (\hat{\boldsymbol{v}}_{\boldsymbol{so}} \cdot \hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{n}}_{so} \cdot \hat{\boldsymbol{n}}_i)/\overline{D}_1 \tag{G.17}$$

where

$$D_1 = q_z D_1/|q_z| \tag{G.18}$$

$$\begin{aligned}\hat{\boldsymbol{v}}_{so}\cdot(\hat{\boldsymbol{n}}_1\times\hat{\boldsymbol{t}}) &= (\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_i)(1-(\hat{\boldsymbol{n}}_i\cdot\hat{\boldsymbol{n}}_s))k_1/(qD_1)\\ &= (\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_i)q/(2k_1D_1)\end{aligned} \tag{G.19}$$

$$\begin{aligned}\hat{\boldsymbol{h}}_{so}\cdot(\hat{\boldsymbol{n}}_1\times\hat{\boldsymbol{t}}) &= (\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_i)(1-(\hat{\boldsymbol{n}}_i\cdot\hat{\boldsymbol{n}}_s))k_1/(qD_1)\\ &= (\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_i)q/(2k_1D_1)\end{aligned} \tag{G.20}$$

with the above vector identities (G.2) and (G.3) can be expressed in $\hat{\boldsymbol{n}}_i$ and $\hat{\boldsymbol{n}}_{so}$. For horizontally polarized incident wave

$$\begin{aligned}\hat{\boldsymbol{v}}_{so}\cdot(\hat{\boldsymbol{n}}_1\times\boldsymbol{E}) &= \left((1+R_\perp)(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{t}})(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_1\times\hat{\boldsymbol{t}})-(1-R_\parallel)(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{d}})(\hat{\boldsymbol{n}}_1\cdot\hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{t}})\right)E_o\\ &= \frac{qE_o}{2k_1D_1\overline{D}_1}\left\{(1+R_\perp)(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{so})(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_i)-(1-R_\parallel)(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{so})(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_i)\right\}\end{aligned} \tag{G.21}$$

$$\begin{aligned}\hat{\boldsymbol{h}}_{so}\cdot(h_1\hat{\boldsymbol{n}}_1\times\boldsymbol{H}) &= -\left((1-R_\perp)(\hat{\boldsymbol{n}}_1\cdot\hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{t}})(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{t}})+(1+R_\parallel)(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{d}})(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_1\times\hat{\boldsymbol{t}})\right)E_o\\ &= \frac{qE_o}{2k_1D_1\overline{D}_1}\left\{-(1-R_\perp)(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{so})+(1+R_\parallel)(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{so})\right\}\end{aligned} \tag{G.22}$$

Equation (5.33) for ${}^{o}E^{s}_{hh}$ can be found from (G.20) and (G.21), since

$$\begin{aligned}\hat{\boldsymbol{h}}_{so}\cdot\boldsymbol{E}^s &= KI_1\{(G.20)+(G.21)\}\\ &= M_1\left\{R_\parallel(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{so})+R_\perp(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{so})\right\}\end{aligned} \tag{G.23}$$

where

$$M_1 = KI_1E_oq/(k_1D_1\overline{D}_1) \tag{G.24}$$

To compute ${}^{o}E^{s}_{vh}$, we need the following vector products:

$$\begin{aligned}\hat{\boldsymbol{v}}_{so}\cdot(h_1\hat{\boldsymbol{n}}_1\times\boldsymbol{H}) &= -\left((1-R_\perp)(\hat{\boldsymbol{n}}_1\cdot\hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{t}})(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{t}})+(1+R_\parallel)(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{d}})(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_1\times\hat{\boldsymbol{t}})\right)E_o\\ &= \frac{qE_o}{2k_1D_1\overline{D}_1}\left\{(1-R_\perp)(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{so})+(1+R_\parallel)(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{so})\right\}\end{aligned} \tag{G.25}$$

$$\begin{aligned}\hat{\boldsymbol{h}}_{so}\cdot(\hat{\boldsymbol{n}}_1\times\boldsymbol{E}) &= \left((1+R_\perp)(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{t}})(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_1\times\hat{\boldsymbol{t}})-(1-R_\parallel)(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{d}})(\hat{\boldsymbol{n}}_1\cdot\hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{t}})\right)E_o\\ &= \frac{qE_o}{2k_1D_1\overline{D}_1}\left\{(1+R_\perp)(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{so})(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_i)+(1-R_\parallel)(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{so})(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_i)\right\}\end{aligned} \tag{G.26}$$

In view of (G.3)

$$
\begin{aligned}
{}^{o}E_{vh}^{s} &= \hat{\boldsymbol{v}}_{so} \cdot \boldsymbol{E}^{s} \\
&= M_1 \left\{ R_{\parallel} \left(\hat{\boldsymbol{v}}_{so} \cdot \hat{\boldsymbol{n}}_i \right) \left(\hat{\boldsymbol{h}} \cdot \hat{n}_{so} \right) - R_{\perp} \left(\hat{\boldsymbol{h}}_{so} \cdot \hat{\boldsymbol{n}}_i \right) \left(\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}_{so} \right) \right\}
\end{aligned} \tag{G.27}
$$

The above equation is the same as (5.34). For vertically polarized incident wave, ${}^{o}E_{vv}^{s}$ can be obtained from (G.22) by interchanging $\hat{\boldsymbol{v}}$ and $\hat{\boldsymbol{h}}$ as well as $\hat{\boldsymbol{v}}_{\boldsymbol{so}}$ and $\hat{\boldsymbol{h}}_{\boldsymbol{so}}$:

$$
{}^{o}E_{vv}^{s} = M_1 \left\{ R_{\parallel} \left(\hat{\boldsymbol{v}}_{so} \cdot \hat{\boldsymbol{n}}_i \right) \left(\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}_{so} \right) + R_{\perp} \left(\hat{\boldsymbol{h}}_{so} \cdot \hat{\boldsymbol{n}}_i \right) \left(\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{n}}_{so} \right) \right\} \tag{G.28}
$$

Similarly, ${}^{o}E_{hv}^{s}$ can be found from (G.26) with the same interchanges:

$$
{}^{o}E_{hv}^{s} = M_1 \left\{ R_{\parallel} \left(\hat{\boldsymbol{h}}_{so} \cdot \hat{\boldsymbol{n}}_i \right) \left(\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}_{so} \right) - R_{\perp} \left(\hat{\boldsymbol{v}}_{so} \cdot \hat{\boldsymbol{n}}_i \right) \left(\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{n}}_{so} \right) \right\} \tag{G.29}
$$

in (G.22) through (G.28)

$$
R_{\parallel} = \left(k_2 \cos q_o - k_1 \cos f_o \right) / \left(k_2 \cos q_o + k_1 \cos f_o \right) \tag{G.30}
$$

$$
R_{\perp} = \left(k_1 \cos q_o - k_2 \cos f_o \right) / \left(k_1 \cos q_o + k_2 \cos f_o \right) \tag{G.31}
$$

where f_o is the local angle of transmission. The dot products in the field expressions are given below:

$$
\hat{\boldsymbol{v}}_{so} \cdot \hat{\boldsymbol{n}}_i = \sin q_o \cos q_{so} \cos\left(f_{so} - f_o \right) + \cos q_o \sin q_{so} \tag{G.32}
$$

$$
\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}_{so} = \cos q_o \sin q_{so} \cos\left(f_{so} - f_o \right) + \sin q_o \cos q_{so} \tag{G.33}
$$

$$
\hat{\boldsymbol{h}}_{so} \cdot \hat{\boldsymbol{n}}_i = -\sin q_o \sin\left(f_{so} - f_o \right) \tag{G.34}
$$

$$
\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{n}}_{so} = \sin q_{so} \sin\left(f_{so} - f_o \right) \tag{G.35}
$$