Appendix H

Derivation of the Scattered Fields

in the Medium 2 at Burnt Coal Seam and Peat Interface

From (5.45), the vector nature of ${}^{o}\boldsymbol{E}^{s}$ is characterized by

$$\begin{aligned} {}^{o}\boldsymbol{E}_{n}^{s} &\equiv \boldsymbol{E}^{s}/KI_{1} \\ &= \hat{\boldsymbol{n}}_{so}^{t}\times\left(\hat{\boldsymbol{n}}_{2}\times\boldsymbol{E}-\hat{\boldsymbol{n}}_{so}^{t}\times\left(h_{2}\hat{\boldsymbol{n}}_{1}\times\boldsymbol{H}\right)\right) \\ &= \hat{\boldsymbol{n}}_{so}^{t}\times\left(\hat{\boldsymbol{n}}_{2}\times\boldsymbol{E}\right)+\left(h_{2}\hat{\boldsymbol{n}}_{2}\times\boldsymbol{H}\right)-\left(\hat{\boldsymbol{n}}_{so}^{t}\cdot\left(h_{2}\hat{\boldsymbol{n}}_{2}\times\boldsymbol{H}\right)\right)\hat{\boldsymbol{n}}_{so}^{t} \end{aligned} \tag{H.1}$$

The polarization factors are

$$\hat{\boldsymbol{h}}_{so}\cdot{}^{o}\boldsymbol{E}_{n}^{s}=\hat{\boldsymbol{v}}_{so}\cdot\left(\hat{\boldsymbol{n}}_{2}\times\boldsymbol{E}\right)+\hat{\boldsymbol{h}}_{so}\cdot\left(h_{2}\hat{\boldsymbol{n}}_{2}\times\boldsymbol{H}\right) \tag{H.2}$$

$$\hat{\boldsymbol{v}}_{so}\cdot{}^{o}\boldsymbol{E}_{n}^{s}=-\hat{\boldsymbol{h}}_{so}\cdot\left(\hat{\boldsymbol{n}}_{2}\times\boldsymbol{E}\right)+\hat{\boldsymbol{v}}_{so}\cdot\left(h_{\boldsymbol{2}}\hat{\boldsymbol{n}}_{2}\times\boldsymbol{H}\right) \tag{H.3}$$

Under the stationary-phase approximation, $\hat{\boldsymbol{n}}_{1}$, $\hat{\boldsymbol{t}}$, $\hat{\boldsymbol{d}}$ can be expressed in terms of propagation vectors, $\hat{\boldsymbol{n}}_{so}$ and $\hat{\boldsymbol{n}}_{i}$, as in Appendix G.

$$\hat{\boldsymbol{n}}_{1}=\frac{\left(\hat{\boldsymbol{x}}\bar{q}_{x}+\hat{\boldsymbol{y}}\bar{q}_{y}+\hat{\boldsymbol{z}}\bar{q}_{z}\right)\left|\bar{q}_{z}\right|}{\bar{q}_{z}\bar{q}}=\frac{\left(k_{2}\hat{\boldsymbol{n}}_{so}-k_{1}\hat{\boldsymbol{n}}_{i}\right)\left|\bar{q}_{z}\right|}{\bar{q}\bar{q}_{z}} \tag{H.4}$$

where

$$\bar{q}^{2}=\bar{q}_{x}^{2}+\bar{q}_{y}^{2}+\bar{q}_{z}^{2}=k_{1}^{2}+k_{2}^{2}-2k_{1}k_{2}\left(\hat{\boldsymbol{n}}_{so}\cdot\hat{\boldsymbol{n}}_{i}\right) \tag{H.5}$$

$$\hat{\boldsymbol{t}}=\frac{\left(\hat{\boldsymbol{n}}_{1}\times\hat{\boldsymbol{n}}_{so}\right)\left|q_{z}\right|}{\bar{q}_{z}D_{2}} \tag{H.6}$$

$$D_{2}=\left|\hat{\boldsymbol{n}}_{i}\times\hat{\boldsymbol{n}}_{so}\right|=\left|\hat{\boldsymbol{n}}_{i}\times\left(\hat{\boldsymbol{h}}_{so}\times\hat{\boldsymbol{v}}_{so}\right)\right|=\left|\left(\hat{\boldsymbol{n}}_{i}\cdot\hat{\boldsymbol{v}}_{so}\right)^{2}+\left(\hat{\boldsymbol{n}}_{i}\cdot\hat{\boldsymbol{h}}_{so}\right)^{2}\right|^{\frac{1}{2}} \tag{H.7}$$

$$\hat{\boldsymbol{d}}=\hat{\boldsymbol{n}}_{\boldsymbol{i}}\times\hat{\boldsymbol{t}}=\left(\left(\hat{\boldsymbol{n}}_{i}\cdot\hat{\boldsymbol{n}}_{so}\right)\hat{\boldsymbol{n}}_{i}-\hat{\boldsymbol{n}}_{so}\right)\left|\bar{q}_{z}\right|/\left(\bar{q}_{z}D_{2}\right) \tag{H.8}$$

with these local coordinate vectors expressed in $\hat{\boldsymbol{n}}_i$ and $\hat{\boldsymbol{n}}_{so}$, all other vector products can be expressed in $\hat{\boldsymbol{n}}_i$ and $\hat{\boldsymbol{n}}_{so}$

$$\hat{\boldsymbol{n}}_2 \times \hat{\boldsymbol{t}} = \frac{\{k_1(\hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{n}}_{so}) - k_2\}\hat{\boldsymbol{n}}_i + \{k_2(\hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{n}}_{so}) - k_1\}\hat{\boldsymbol{n}}_{so}}{\overline{q} D_2} \tag{H.9}$$

$$\hat{\boldsymbol{n}}_2 \cdot \hat{\boldsymbol{n}}_{so} = -(k_2 - k_1(\hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{n}}_{so}))|\overline{q}_z| / \overline{q}\overline{q}_z = \hat{\boldsymbol{n}}_2 \cdot \hat{\boldsymbol{n}}_t \equiv \cos f_o \tag{H.10}$$

$$\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{t}} = (\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}_{so}) / \overline{D}_2 \tag{H.11}$$

$$\hat{\boldsymbol{h}}_{so} \cdot \hat{\boldsymbol{t}} = -(\hat{\boldsymbol{v}}_{so} \cdot \hat{\boldsymbol{n}}_i) / \overline{D}_2 \tag{H.12}$$

$$\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{t}} = -(\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{n}}_{so}) / \overline{D}_2 \tag{H.13}$$

$$\hat{\boldsymbol{v}}_{so} \cdot \hat{\boldsymbol{t}} = -(\hat{\boldsymbol{h}}_{so} \cdot \hat{\boldsymbol{n}}_i) / \overline{D}_2 \tag{H.14}$$

$$\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{d}} = -(\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{n}}_{so}) / \overline{D}_2 \tag{H.15}$$

$$\hat{\boldsymbol{h}}_{so} \cdot \hat{\boldsymbol{d}} = (\hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{n}}_{so})(\hat{\boldsymbol{h}}_{so} \cdot \hat{\boldsymbol{n}}_i) / \overline{D}_2 \tag{H.16}$$

$$\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{d}} = -(\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}_{so}) / \overline{D}_2 \tag{H.17}$$

$$\hat{\boldsymbol{v}}_{so} \cdot \hat{\boldsymbol{d}} = (\hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{n}}_{so})(\hat{\boldsymbol{v}}_{so} \cdot \hat{\boldsymbol{n}}_i) / \overline{D}_2 \tag{H.18}$$

where

$$D_2 = \overline{q}_z D_2 / |\overline{q}_z| \tag{H.19}$$

$$\hat{\boldsymbol{v}}_{so} \cdot (\hat{\boldsymbol{n}}_2 \times \hat{\boldsymbol{t}}) = (\hat{\boldsymbol{v}}_{so} \cdot \hat{\boldsymbol{n}}_i)(k_1(\hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{n}}_{so}) - k_2) / (\overline{q} D_2) \tag{H.20}$$

$$\hat{\boldsymbol{h}}_{so} \cdot (\hat{\boldsymbol{n}}_2 \times \hat{\boldsymbol{t}}) = (\hat{\boldsymbol{h}}_{so} \cdot \hat{\boldsymbol{n}}_i)(k_1(\hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{n}}_{so}) - k_2) / (\overline{q} D_1) \tag{H.21}$$

with the above vector identities (H.2) and (H.3) can be expressed in $\hat{\boldsymbol{n}}_i$ and $\hat{\boldsymbol{n}}_{so}$. For horizontally polarized incident wave

$$\hat{\boldsymbol{v}}_{so}\cdot(\hat{\boldsymbol{n}}_2\times\boldsymbol{E}) = \left(T_\perp(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{t}})(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_2\times\hat{\boldsymbol{t}}) - T_\parallel(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{d}})(\hat{\boldsymbol{n}}_2\cdot\hat{\boldsymbol{n}}_t)(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{t}})h_2/h_1\right)E_o$$
$$= -\{k_2 - k_1(\hat{\boldsymbol{n}}_i\cdot\hat{\boldsymbol{n}}_{so})\}E_o\times\frac{T_\perp(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{so})(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_i) + T_\parallel(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{so})(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_i)h_2/h_1}{\overline{q}D_2\overline{D}_2} \tag{H.22}$$

$$\hat{\boldsymbol{h}}_{so}\cdot(h_2\hat{\boldsymbol{n}}_2\times\boldsymbol{H}) = -\left(T_\perp(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{t}})(\hat{\boldsymbol{n}}_1\cdot\hat{\boldsymbol{n}}_t)(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{t}}) + T_\parallel(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{d}})(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_2\times\hat{\boldsymbol{t}})h_2/h_1\right)E_o$$
$$= \text{(H.22)} \tag{H.23}$$

$$\hat{\boldsymbol{v}}_{so}\cdot(h_2\hat{\boldsymbol{n}}_2\times\boldsymbol{H}) = -\left(T_\perp(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{t}})(\hat{\boldsymbol{n}}_2\cdot\hat{\boldsymbol{n}}_t)(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{t}}) + T_\parallel(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{d}})(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_2\times\hat{\boldsymbol{t}})h_2/h_1\right)E_o$$
$$= \{k_2 - k_1(\hat{\boldsymbol{n}}_i\cdot\hat{\boldsymbol{n}}_{so})\}E_o\times\frac{T_\perp(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{so})(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_i) - T_\parallel(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{so})(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_i)h_2/h_1}{\overline{q}D_2\overline{D}_2} \tag{H.24}$$

$$\hat{\boldsymbol{h}}_{so}\cdot(\hat{\boldsymbol{n}}_2\times\boldsymbol{E}) = \left(T_\perp(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{t}})(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_2\times\hat{\boldsymbol{t}}) - T_\parallel(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{d}})(\hat{\boldsymbol{n}}_2\cdot\hat{\boldsymbol{n}}_t)(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{t}})h_2/h_1\right)E_o$$
$$= -\text{(H.24)} \tag{H.25}$$

By substituting (H.22) through (H.25) into (H.2) and (H.3), we obtain

$${}^oE^s_{hh} = KI_2\left(\hat{\boldsymbol{h}}_{so}\cdot{}^o\boldsymbol{E}^s_n\right)$$
$$= -M_2\left\{T_\perp(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{so})(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_i) + T_\parallel(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{so})(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_i)h_2/h_1\right\} \tag{H.26}$$

$${}^oE^s_{vh} = KI_2\left(\hat{\boldsymbol{v}}_{so}\cdot{}^o\boldsymbol{E}^s_n\right)$$
$$= M_2\left\{T_\perp(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{h}}\cdot\hat{n}_{so}) - T_\parallel(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{so})h_2/h_1\right\} \tag{H.27}$$

where $M_2 = 2KI_2E_o\left(k_2 - k_1(\hat{\boldsymbol{n}}_i\cdot\hat{\boldsymbol{n}}_{so})\right)/\left(\overline{q}D_2\overline{D}_2\right)$. For a vertically polarized incident wave, ${}^oE^s_{vv}$ and ${}^oE^s_{hv}$ can be obtained from (H.26) and (H.27) respectively by interchanging $\hat{\boldsymbol{v}}$ with $\hat{\boldsymbol{h}}$, and $\hat{\boldsymbol{v}}_{so}$ with $\hat{\boldsymbol{h}}_{so}$

$${}^oE^s_{vv} = -M_2\left\{T_\perp(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{so}) + T_\parallel(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{so})h_2/h_1\right\} \tag{H.28}$$

$${}^oE^s_{hv} = M_2\left\{T_\perp(\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{so}) - T_\parallel(\hat{\boldsymbol{h}}_{so}\cdot\hat{\boldsymbol{n}}_i)(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{so})h_2/h_1\right\} \tag{H.29}$$

The dot products in the field expressions are given by

$$\hat{\boldsymbol{v}}_{so}\cdot\hat{\boldsymbol{n}}_i = -\sin q_o\cos q^t_{so}\cos(f_{so} - f_o) + \cos q_o\sin q^t_{so} \tag{H.30}$$

$$\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{so} = -\cos q_o\sin q^t_{so}\cos(f_{so} - f_o) + \sin q_o\cos q^t_{so} \tag{H.31}$$

$$\hat{\boldsymbol{h}}_{so} \cdot \hat{\boldsymbol{n}}_{i} = -\sin q_{o} \sin\left(f_{so} - f_{o}\right) \tag{H.32}$$

$$\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{n}}_{so} = \sin q_{so}^{t} \sin\left(f_{so} - f_{o}\right) \tag{H.33}$$