Appendix A

Analysis of Scattered Waves on Two Layers of Tree Trunk

(TM mode)

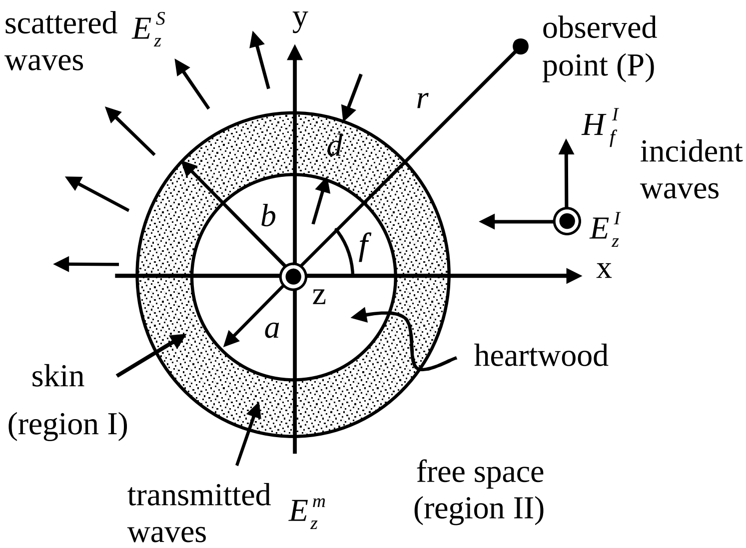

Figure A.1. Geometry of analysis of scattered TM mode wave (two layers)

The coordinate system for analysis and the direction of incident electromagnetic fields are shown in figure A.1. An infinitely cylindrical heartwood of radius *a* is coated with the skin (bark). The water content of heartwood is high. Consequently, heartwood may be assumed to be an infinite length of perfect conductor or electromagnetic fields in heartwood is zero. This skin has dielectric constant e_r and specific permeability m_r (=1+*j*0) and its thickness is *d*. The axis of the cylinder is along the z-axis and the incident wave is propagating in the direction of –x axis. The electric fields are given by the following equations in region I (field in skin layer) and region II (fields in free space) for vertical polarization or TM mode.

Incident wave $$E_z^I = E_o^I \exp(jk_o x) = E_o^I \sum_{m=0}^{\infty} U_m J_m(k_o r) j^m \cos mf \qquad (r > b) \qquad \text{(A.1)}$$

Scattered wave $E_z^S = E_o^I \sum_{m=0}^{\infty} b_m H_m^{(2)}(k_o r)\cos mf$ $(r > b)$ (A.2)

Transmitted wave $E_z^m = E_o^I \sum_{m=0}^{\infty} \{a_m J_m(kr) + a'_m N_m(kr)\}\cos mf$ $(a < r \le b)$ (A.3)

where the wave number of skin is $k = k_o\sqrt{m_r e_r}$ and k_o is wave number in free space. a_m to b_m are the amplitude coefficients. E_o^I is amplitude coefficient of incident electric field. J_m, N_m, and $H_m^{(2)}$ are m-th order of Bessel function, Neumann function, and 2nd kind of Hankel function. Then

$$U_m = \begin{cases} 1 & (m = 0) \\ 2 & (m = 1,2,3,\cdots) \end{cases} \quad \text{(A.4)}$$

and $m = 0, 1, 2, 3, \cdots$. By substituting (A.1) to (A.3) into Maxwell's equations below

$$\nabla \times \boldsymbol{E} = -m\frac{\partial \boldsymbol{H}}{\partial t} \quad \text{(A.5)}$$

the magnetic field of each medium was derived as

$$H_f^I = \frac{k_o E_o^I}{jwm_o}\sum_{m=0}^{\infty} U_m J'_m(k_o r) j^m \cos mf \quad (r > b) \quad \text{(A.6)}$$

$$H_f^S = \frac{k_o E_o^I}{jwm_o}\sum_{m=0}^{\infty} b_m H_m^{(2)\prime}(k_o r)\cos mf \quad (r > b) \quad \text{(A.7)}$$

$$H_f^m = \frac{k E_o^I}{jwm}\sum_{m=0}^{\infty} \{a_m J'_m(kr) + a'_m N'_m(kr)\}\cos mf \quad (a < r \le b) \quad \text{(A.8)}$$

Further, by substituting (A.1) to (A.3) and (A.6) to (A.8) into the boundary condition of each interface between media given below:

$r = a$ $E_z^m = 0$ (A.9)

$r = b$ $H_f^m = H_f^I + H_f^S$ (A.10)

$E_z^m = E_z^I + E_z^S$ (A.11)

the amplitude coefficient b_m of scattered wave from tree trunk E_f^S was obtained as;

$$b_m = -\frac{U_m j^m \{aZ_r J'_m(k_o b) - bJ_m(k_o b)\}}{aZ_r H'^{(2)}_m(k_o b) - bH^{(2)}_m(k_o b)} \tag{A.12}$$

where

$$Z_r = \sqrt{m_r / e_r} \tag{A.13}$$

$$a = N_m(kb)J_m(ka) - J_m(kb)N_m(ka) \tag{A.14}$$

$$b = N'_m(kb)J_m(ka) - J'_m(kb)N_m(ka) \tag{A.15}$$

Finally, by substituting the amplitude coefficient b_m of (A.12) into (A.2), the scattered electric field is obtained.