Appendix B

Analysis of Scattered Waves on Three Layers of Tree Trunk

(TM mode)

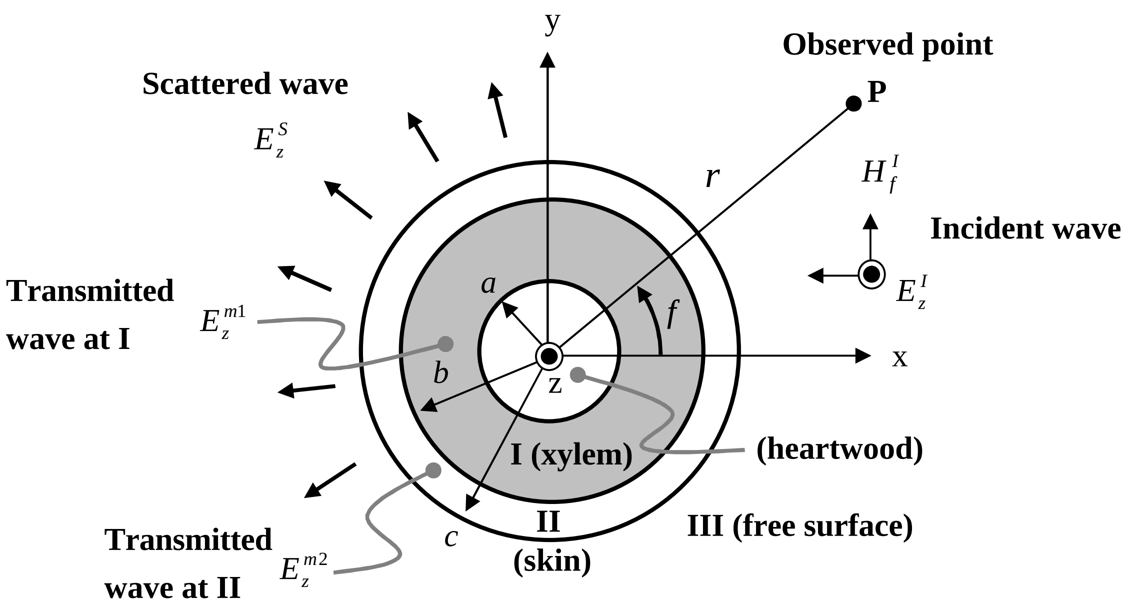

Figure B.1. Geometry of analysis of scattered TM mode wave (three layers)

The two-dimensional model of tree trunk is shown in figure B.1. Three layers of media compose this model of tree trunk with infinite length in z-axis. The radii of heartwood, xylem and skin layer are *a*, *b*, and *c*, respectively. The properties of xylem and skin are determined by e_{r1}, m_{r1} and e_{r2}, m_{r2}, respectively. Where, e_{ri} and m_{ri} (i=1,2) are complex dielectric constant and complex permeability, respectively. The water content of heartwood is high, consequently, heartwood may be assumed to be an infinite length of perfect conductor or electromagnetic fields in heartwood is zero. Here, incident wave is assumed as a plane wave that has transverse magnetic (TM) mode and incident angle f with respect to direction of observed point P from origin of coordinate. This wave propagates is –x direction. Based on this figure, the z component of the electromagnetic fields in free space, xylem and skin are determined as

Incident wave $$E_z^I = E_o^I \exp(jk_o r) = E_o^I \sum_{m=0}^{\infty} U_m J_m(k_o r) j^m \cos mf \qquad (r > c) \qquad \text{(B.1)}$$

Scattered wave $$E_z^S = E_o^I \sum_{m=0}^{\infty} b_m H_m^{(2)}(k_o r) \cos mf \qquad (r > c) \qquad \text{(B.2)}$$

Medium I $$E_z^{m1} = E_o^I \sum_{m=0}^{\infty} \{a_{1m} J_m(k_1 r) + a'_{1m} N_m(k_1 r)\} \cos mf \qquad (a < r \le b) \qquad \text{(B.3)}$$

Medium II $$E_z^{m2} = E_o^I \sum_{m=0}^{\infty} \{a_{2m} J_m(k_2 r) + a'_{2m} N_m(k_2 r)\} \cos mf \qquad (b < r \le c) \qquad \text{(B.4)}$$

Where the wave numbers of each medium are $k_1 = k_o \sqrt{m_{r1} e_{r1}}$ and $k_2 = k_o \sqrt{m_{r2} e_{r2}}$, and k_o is wave number in free space. a_{1m} to b_m are amplitude coefficients. E_o^I is amplitude coefficient of incident electric field. J_m, N_m, and $H_m^{(2)}$ are *m*-th of Bessel function, Neumann function, and 2nd kind of Hankel function, respectively. Then $m = 0, 1, 2, 3, \cdots$ and

$$U_m = \begin{cases} 1, & m = 0 \\ 2, & m = 1, 2, \cdots \end{cases} \qquad \text{(B.5)}$$

By substituting (B.1) to (B.4) in Maxwell's equations below

$$\nabla \times \boldsymbol{E} = -m \frac{\partial \boldsymbol{H}}{\partial t} \qquad \text{(B.6)}$$

the magnetic field of each medium is derived as

Incident wave $$H_f^I = \frac{k_o E_o^I}{jwm_o} \sum_{m=0}^{\infty} U_m J'_m(k_o r) j^m \cos mf \qquad (r > b) \qquad \text{(B.7)}$$

Scattered wave $H_f^S = \frac{k_o E_o^I}{jwm_o} \sum_{m=0}^{\infty} b_m H_m^{(2)\prime}(k_o r) \cos mf$ $\quad (r > b)$ (B.8)

Medium I $H_f^{m1} = \frac{k_1 E_o^I}{jwm_1} \sum_{m=0}^{\infty} \{a_{1m} J'_m(k_1 r) + a'_{1m} N'_m(k_1 r)\} \cos mf$ $\quad (c < r \le a)$ (B.9)

Medium II $H_f^{m2} = \frac{k_2 E_o^I}{jwm_2} \sum_{m=0}^{\infty} \{a_{2m} J'_m(k_2 r) + a'_{2m} N'_m(k_2 r)\} \cos mf$ $\quad (a < r \le b)$ (B.10)

Further, by substituting (B.1) to (B.4) and (B.7) to (B.10) into the boundary condition of each interface between media given below:

$$r = a \qquad E_z^{m1} = 0 \tag{B.11}$$

$$r = b \qquad H_f^{m1} = H_f^{m2} \tag{B.12}$$

$$E_z^{m1} = E_z^{m2} \tag{B.13}$$

$$r = c \qquad H_f^{m2} = H_f^S + H_f^I \tag{B.14}$$

$$E_z^{m2} = E_z^S + E_z^I \tag{B.15}$$

the amplitude coefficient b_m of scattered wave from tree trunk E_f^S is obtained as;

$$b_m = \frac{U_m j^m \{a_{6m} J'_m(k_o c) - a_{7m} J_m(k_o c)\}}{a_{7m} H_m^{(2)}(k_o c) - a_{6m} H_m^{(2)\prime}(k_o c)} \tag{B.16}$$

where

$$a_{7m} = \frac{k_2 m_o}{k_o m_2} \left\{ N'_m(k_2 c) - \frac{a_{5m}}{a_{4m}} J'_m(k_2 c) \right\} \tag{B.17}$$

$$a_{6m} = N_m(k_2 c) - \frac{a_{5m}}{a_{4m}} J_m(k_2 c) \tag{B.18}$$

$$a_{5m} = a_{3m} \frac{k_1}{m_1} N_m(k_2 b) - a_{2m} \frac{k_2}{m_2} N'_m(k_2 b) \tag{B.19}$$

$$a_{4m} = a_{3m} \frac{k_1}{m_1} J_m(k_2 b) - a_{2m} \frac{k_2}{m_2} J'_m(k_2 b) \tag{B.20}$$

$$a_{3m} = N'_m(k_1 b) - a_{1m} J'_m(k_1 b) \tag{B.21}$$

$$a_{2m} = N_m(k_1 b) - a_{1m} J_m(k_1 b) \tag{B.22}$$

$$a_{1m} = \frac{N_m(k_1 a)}{J_m(k_1 a)} \tag{B.23}$$

Finally, by substituting the amplitude coefficient b_m of (B.16) into (B.2), the scattered electric field is obtained.